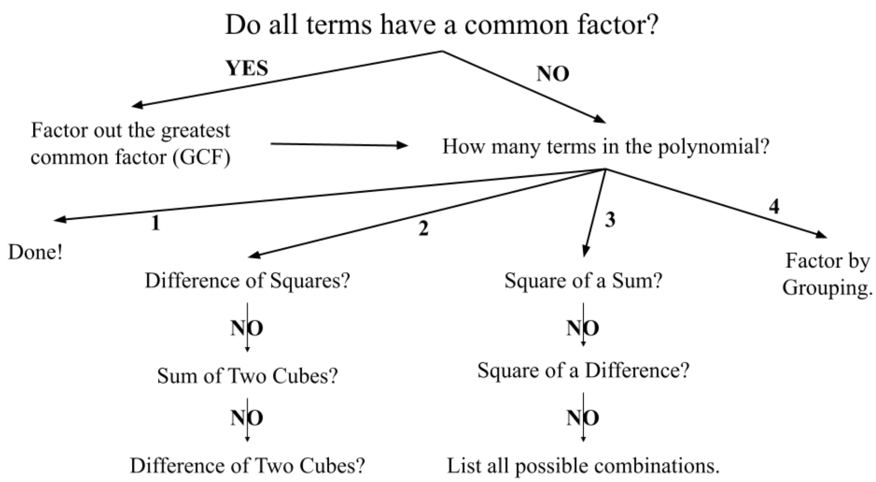
Math 1320: Factoring Strategies

What is factoring? When we factor polynomials, we are finding an equivalent expression that is a product of factors. A prime factor of a polynomial is one that cannot be factored into polynomials of a smaller degree. When a polynomial is expressed as a product of prime factors, we say that the polynomial has been factored completely. Below is an example of a factored polynomial:

$$2x^2 + x - 6 \quad = \quad (2x-3)(x+2)$$

Notice that each of the factors, $(2x - 3)(x + 2)$, are prime factors. There is no common factor of the terms or a special property that we can use to factor further. Factoring can be complicated and is not an exact science. Let's look at steps that make the process more manageable.

Strategy for Factoring a Polynomial



1. If there is a common factor, factor out the Greatest Common Factor (GCF).

2. Determine the number of terms in the polynomial and try factoring as follows:

   - If there are two terms, can the binomial be factored using one the following special rules?
     - Difference of two squares: $A^2 - B^2 = (A+B)(A-B)$
     - Sum of two cubes: $A^3 + B^3 = (A+B)(A^2 - AB + B^2)$
     - Difference of two cubes: $A^3 - B^3 = (A-B)(A^2 + AB + B^2)$
   - If there are three terms, check if the trinomial is a perfect square trinomial. If so, factor by using one of the special rules:

     $$A^2 + 2AB + B^2 = (A+B)^2 \qquad\qquad A^2 - 2AB + B^2 = (A-B)^2$$

     If the trinomial is not a perfect square, try factoring by trial and error, the box method, or the diamond method.
   - If there are four or more terms, try factoring by grouping.

3. For every factor found, repeat the process to make sure it is a prime factor.

4. To check that you factored the polynomial correctly, multiply the factors using the distribution property. If the product is equal to the original polynomial, the factorization is correct.

Example 1. Factoring Out GCF
The GCF ($9x^2$ in the example below) of a polynomial is the expression of the highest degree that divides each term of the polynomial.

$$ab + ac = a(b + c)$$

$$18x^3 + 27x^2 \quad \rightarrow \quad 9x^2(2x) + 9x^2(3) \quad \rightarrow \quad 9x^2(2x + 3)$$

Looking at the polynomial above, I can see that both $18x^3$ and $27x^2$ can be divided by $9x^2$. Then, I will rewrite the terms as a product with $9x^2$.

Example 2. Factoring by Grouping
This method is used when there are four or more terms in the polynomial and we can group terms in pairs that have GCFs.

$$\begin{aligned} 6x^3 + 3x^2 - 12x - 6 \quad &\rightarrow (6x^3 + 3x^2) + (-12x - 6) && \text{Group factors into pairs} \\ &\rightarrow 3x^2(2x + 1) + -6(2x + 1) && \text{Factor out GCF of each set of parentheses} \\ &\rightarrow (3x^2 - 6)(2x + 1) && \text{Factor GCF, } (2x + 1) \end{aligned}$$

⋆ Note that the terms in the parentheses are the same [$(2x + 1)$], this must always be the case when factoring by grouping.

Example 3. Difference of Two Squares

$$x^2 - 16$$

The polynomial above has no GCF and there are only two terms. The terms do have one thing in common, they are both squares. Since the squares are being subtracted, I will apply the difference of two squares property:

$$\begin{aligned} x^2 - 16 \quad &\rightarrow (x)^2 - (4)^2 && \text{Rewrite in terms of } A^2 + B^2 \\ &\rightarrow (x + 4)(x - 4) && \text{Apply difference of squares formula} \end{aligned}$$